The psychology of conditionals

Class 1

Introduction

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Course description

Conditionals - indicative, counterfactual, deontic, and others - are central to the study of reasoning in logic, philosophy, and psychology.

New Bayesian approaches to the study of cognition have had a major impact on the psychology of reasoning generally and the psychology of conditional reasoning in particular.

These classes will introduce the new psychological accounts of conditionals, grounded in Bayesian probability theory.

An example of a traditional study in the psychology of reasoning

There are five blocks in a stack. The second one from the top is green, and the fourth is not green. Is a green block directly on top of a non-green block?

(A) Yes   (B) No   (C) Cannot tell.

The answer to this question is a definite choice, not a probability judgment.

See Gilio & Over (2012) on this problem.

An example in which a probability judgment is necessary

A TV presenter tells a contestant on a game show that there is a prize behind door a or door b.

The contestant infers, "If the prize is not behind a then it is behind b". Is this inference correct?

Johnson-Laird & Byrne (2002) claim everyone will say, "Yes". But the TV presenter has placed the prize behind a, and what does she infer, with what degree of belief?

See again Gilio & Over (2012).

Assumptions and belief

Traditional psychology of reasoning was assumption-based and binary. Bayesian approaches are belief-based and probabilistic.

In the tradition, participants were asked to assume that given premises were true and then to state what necessarily followed.

Bayesian approaches note that almost all inference in everyday life and science is from uncertain premises. Such inferences are from degrees of belief to degrees of belief in a dynamic process of belief revision and updating.

Conditionals are central to reasoning and decision making

Conditionals are at the heart of the psychology of reasoning. Why is that?

Some philosophers have argued that a conditional, *if p then q*, is nothing more than an "inference ticket" for inferring *q* from *p* (Bennett, 2003, pp. 118-119).

A conditional can be expressed as an inference. An inference can be "summed up" as a conditional.

Conditionals are also everywhere in decision making, where the question is "If we take the action, what are the consequences?"

Examples of conditionals

If the fire alarm is going off, we are leaving the building.

If the fire alarm goes off, we will leave the building.

If the fire alarm were to go off, we would leave the building.

If the fire alarm had gone off, we would have left the building.

If the fire alarm goes off, we should leave the building.

Note how "probably" can qualify all of the above.

Wason & Johnson-Laird (1972)

Wason & Johnson-Laird (1972) long set the pattern for traditional psychology of reasoning.

The topics covered in the book were fundamental, but narrow: propositional reasoning and syllogisms. There was something on testing a hypothesis, but surprisingly this was not about confirmation or probability.

The presupposed normative standard for conditionals was <u>truth functional logic</u> and its conditional.

The material conditional

The conditional in elementary truth functional logic is the material conditional. It is logically equivalent to *not-p or q*.

A material conditional, *if p then q*, is true when *p* is true and *q* is true, false when *p* is true and *q* is false, and true when *p* is false.

Its truth value is thus a function, in the strict sense, of the truth values of *p* and *q*. The material conditional can also be called the truth functional conditional.

The truth table for (*not-p or q*)
1 = true, and 0 = false

p \ *q*	1	0
1	1	0
0	1	1

The material conditional ($p \supset q$)
1 = true, and 0 = false

p \ q	1	0
1	1	0
0	1	1

Inferring the material *if* from *or*

A TV presenter tells a contestant on a game show that there is a prize behind door a or door b.

The contestant infers, (*not-a* $\supset$ *b)*.

This is equivalent to (*not-not-a or b)*, and so (*a or b)*.

But how then can the TV presenter come to another conclusion?

The "paradoxes"

Holding that conditionals in natural language are material results in "paradoxes".

<u>First paradox</u>: It is valid to infer *if p then q* from *not-p*.
<u>Second paradox</u>: It is valid to infer *if p then q* from *q*.

The following is a valid inference for the material conditional.

"We will not buy the lottery ticket.
Therefore, if we buy the lottery ticket, we will win millions."

The material conditional and decision making

"We buy the lottery ticket (b) $\supset$ we will win millions (m)."

Being rational, we are less and less likely to buy the lottery ticket as we reflect more and more on the improbability of winning anything in the lottery. But then, the material conditional above will become more and more probable. In that case, why is it rational to decide not to buy the ticket?

P(not-b) $\leq$ P(not-b or m)

The lottery and conditional probability

"If we buy the lottery ticket (b), we will win millions (m)."

Philosophers have long argued that our degree of belief in conditionals like the above is the conditional probability (Adams, 1975). The subjective probability that we will win millions given that we buy the ticket, P(w|b), is extremely low and stays at that level as we become determined not to buy a ticket.

However, it took psychologists many years to come to the same conclusion.

Mental model theory (1991): Fully explicit models

People's mental models for the natural language conditional are proposed to be equivalent to *not-p or q*:

p	**q**
not-p	**q**
not-p	**not-q**

The remaining, implicit, possibility of *p & not-q* makes the material conditional false.

Why are the mental models equivalent to the material conditional?

"If Arthur is in Edinburgh (*p*), then Carol is in Glasgow (*q*)."

This conditional is true when *p* and *q* are true, false when *p* is true and *q* is false. But suppose *p* is false: is the conditional true or false? "It can hardly be false, and so, since the propositional calculus allows only truth or falsity, it must be true" (Johnson-Laird & Byrne, 1991, p. 7).

The truth functional presupposition of traditional psychology of reasoning is crystal clear here.

Mental model theory (1991): The paradoxes

What does a conditional mean in mental model theory? In Johnson-Laird & Byrne (1991, 2002), it means the models of the material conditional.

But why then do people not endorse the paradoxes as valid? The paradoxes are supposed to "throw semantic information away".

This is a pragmatic attempt to explain away people's responses to the paradoxes.

The lottery example again

"If we buy the lottery ticket, we will win millions."

Being rational, we are less and less likely to buy the lottery ticket as we reflect more and more on the improbability of winning anything in the lottery. But the conditional above will not become more and more probable.

The above point is about rational subjective belief. It is not about what might be misleading in communication.

The "paradoxes" logically imply truth functionality

Johnson-Laird & Byrne (2002) are inconsistent in the claims they make about the conditional and truth functionality.

Let *p* and *q* be true. Then the material conditional is true by the second paradox.

Let *p* be true and *q* false. Then the material conditional is false by mental model theory and almost all other theories.

Let *p* be false, making *not-p* true. The material conditional is then true by the first paradox.

Mental model theory (2015)

Johnson-Laird et al. (2015) revise mental model theory (see Baratgin et al, 2015, for comment).

if p then q **still has the same mental models as *not-p or q*, but the paradoxes and or-introduction are logically invalid, i.e. it is invalid to infer *p or q* from *p*.**

The unique aspect of this hypothesis is that the meanings of *if p then q* and *not-p or q* are still identified, but the paradoxes are invalid because or-introduction is invalid.

Mental model theory (2018)

Khemlani, Byrne, & Johnson-Laird (2018) continue to revise mental model theory.

if not-p then q **has the same mental models as** ***p or q*****. These are:**

(p & q) is possible, & (p & not-q) is possible, & (not-p & q) is possible, & (not-p & not-q) is impossible.

The possibilities are said to be "epistemic", but that implies that there is not an objective concept of truth for ***p or q*****, since epistemic possibility is a subjective notion.**

Conjunction and disjunction

It is easy to prove that the validity of &-elimination implies the validity of or-introduction.

p & q logically implies *p*, consequently *not-p* logically implies *not-(p & q)*, which logically implies *not-p or not-q*.

Using double negation, we also derive that *p* logically implies *p or q* from the validity of &-elimination. Therefore, if or-introduction is invalid, then &-elimination is invalid, and this is highly counter-intuitive.

The or-MP inference

The validity of inferring *r* from the two premises *if p or q then r* and *p* is highly endorsed in experiments (Cruz, Over, & Oaksford, 2017). Its validity depends on the validity of or-introduction.

This two premise inference cannot be a special case because:

If (p or q) then (p or q)
p
Therefore (p or q)

Consistency

Hinterecker, Knauff, & Johnson-Laird (2016) add "valid" inferences to revised mental model theory that are invalid in known modal logics with consistency proofs.

Deleting valid inferences from a system with a consistency proof does not raise questions about the consistency of the new system, but adding supposedly valid inferences does, as the new inferences could lead to a contradiction.

Where then is the consistency proof for revised mental model theory (Oaksford, Over, & Cruz, 2019)?

A look forward

Bayesian reasoning has conditional probability judgments at its very core, for its purpose of belief revision and updating.

The Bayesian turn in cognitive psychology could hardly make more use of human conditional probability judgments, which usually express some degree of uncertainty.

Bayesian psychology of reasoning results from identifying the probability of the conditional with the conditional probability:

$P(\text{if } p \text{ then } q) = P(q|p)$.