Conditionals in Semantic Theory: Truth conditions and logic form

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Topics for today

- Desiderata: some inference patterns
- Some approaches
  - Material conditional
  - (Relativized) strict conditional
  - Variably strict conditional
- Kratzer Semantics for modality and conditionals
  - Premise Semantics
  - Ordering Semantics
  - The restrictor analysis of conditionals
- Conditionals with non-epistemic modals in the consequent

1 Some inference patterns

The following observations about inference patterns are relevant for any analysis of conditionals. It's useful to have handy labels for them because we will look at them again later. The first one, <u>Strengthening the Antecedent</u>, will be of particular interest.

Strengthening the antecedent (Monotonicity)

(1) $$\frac{\textsc{if}(A\,,\,C)}{\textsc{if}(AB\,,\,C)}$$

(2) a. If you strike the match, it will light.
b. ??Therefore, if you strike the match and it is wet, it will light.

Contraposition

(3) $$\frac{\textsc{if}(A\,,\,C)}{\textsc{if}(\neg C\,,\,\neg A)}$$

(4) a. The match is wet. If you strike it, it won't light.
b. ??Therefore, if it lights, you didn't strike it.

Vacuous truth

(5) $$\frac{\neg A}{\textsc{if}(A\,,\,C)}$$

(6) a. You won't strike the match.
b. ??Therefore, if you strike it, it will light
(and if you strike it, it won't light.)

Hypothetical Syllogism (Transitivity)

(7)
$$\frac{\begin{array}{c}\textsc{if}(B\,,\,C)\\ \textsc{if}(A\,,\,B)\end{array}}{\textsc{if}(A\,,\,C)}$$

(8)
a. If I quit my job, I will lose my house.
b. If I win a million, I will quit my job.
c. ??If I win a million, I will lose my house.

Notice: (8) is a counterexample to Transitivity because one can easily hold both premises to be true without endorsing the conclusion.

But oftentimes when instances of this pattern are presented, it's not easy to see this. In particular, it's much harder to see the inference as invalid (i.e., imagine that the premises are true while the conclusion is false) when the order of the premises is reversed:

(9)
a. If I win a million, I will quit my job.
b. If I quit my job, I will lose my house.
c. If I win a million, I will lose my house.

Presumably this is because with this order of the premises, a special interpretation becomes available for the second premise, one which is subordinated (as in modal subordination) to the first premise. Thus the form of the premises is misleading; the inference is really this:

(10)
$$\frac{\begin{array}{c}\textsc{if}(A\,,\,B)\\ \textsc{if}(AB\,,\,C)\end{array}}{\textsc{if}(A\,,\,C)}$$

And this is a pattern that we don't have counterexamples to. This, too, needs to be explained.

2 Truth functionality: the material conditional

For much of history, the dominant approach to conditionals in philosophy has been truth-functional: the truth value of $\textsc{if}(p\,,\,q)$ is a function of the truth values of p and q.

Minimal assumptions imply that if $\textsc{if}(\cdot\,,\,\cdot)$ is to denote a truth-functional sentential connective, it has to be the material conditional (here written '$\rightarrow$', often also '$\supset$'). For a nice argument due to Edgington (1986), see Appendix A.

(11)

p	q	$p \rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1

But it is well-known that the material conditional is a poor rendering of our intuitions about conditinal sentences. Among other problems, it validates all of the above problematic inference patterns.

3 Modality

3.1 Basics

I assume familiarity with basic notions from possible-worlds semantics and modal logic. For details on modal logic, see Appendix B. We can over this if there is interest.

Possible worlds: Complete descriptions of states of affairs; give truth values to all atomic sentences

Note: Linguists typically are not too concerned with ontological questions, such as whether the world we inhabit is among the "possible worlds" in the technical sense, or which possible worlds there are, how many, etc. Linguists also typically sidestep problems of logical omniscience.

Kratzer is influenced by Lewis's realism: all possible worlds exist, and ours is one of them. Others take a more instrumental view: possible worlds are mere tools for modeling things like inference, uncertainty and information, defined to suit one's needs.

Propositions: Sets of possible worlds; denotations of sentences

Truth: Proposition p is true at world w iff $w \in p$.

Consistency: A set Φ of propositions is consistent iff there is some world at which all propositions in Φ are true. (i.e., $\bigcap\Phi \neq \emptyset$)

Proposition p is consistent with a set Φ of propositions iff the result of adding p to Φ is consistent.

(i.e., $\bigcap(\Phi \cup \{p\}) \neq \emptyset$)

Consequence: Proposition p is a consequence of a set Φ of propositions iff p is true at at all worlds at which all propositions in Φ are true. (i.e., $\bigcap\Phi \subseteq p$)

3.2 The strict conditional

- Clarence Irving Lewis (not to be confused with David Kellogg Lewis!) addressed some of the problematic patterns in Lewis (1918).
- For instance, we saw above that the falsehood of the antecedent is sufficient for the truth of the conditional; similarly for the truth of the consequent.
- Lewis introduced the strict conditional as a better way to capture the underlying intuitive notion of "implication": Intuitively, '*if p then q*' states that it is impossible (rather than simply not the case) that p is true while q is false.
- The symbol Lewis used was '$\prec\!\!\!\!3$'; thus '$p \prec\!\!\!\!3\ q$' would be his representation of '*if p then q*' interpreted as the strict conditional. Nowadays one more commonly sees expressions like $\Box(p \to q)$, where '$\to$' is the material conditional and '$\Box$' is the necessity operator familiar from modal logic, here intended with a universal accessibility relation.

Definition 1 (Strict conditional)
'*if p then q*' is true iff for all worlds at which p is true, q is also true.

- The strict conditional is superior to the material conditional:
  - the falsehood of the antecedent is not sufficient for the truth of the conditional;
  - ➻ the falsehood of the conditional does not imply that the antecedent is true.
  - likewise for the truth of the consequent is not sufficient for the truth of the conditional;
  - ➻ the falsehood of the conditional does not imply that the consequent is false.
- BUT the strict conditional still validates Strengthening of the Antecedent, Contraposition, and Transitivity.
- AND the strict conditional is <u>not contingent:</u> it is either true at all worlds or false at all worlds. We cannot model uncertainty as to whether conditionals are true or false (or non-trivial embeddings of conditionals).

- The last two points above should be relatively straightforward to verify. They have to do with the fact that the conditional is interpreted by universal quantification over all worlds.
- The last point above is addressed by introducing a world-dependent domain restriction on the modal operator. In modal logic, this is done by introducing an <u>accessibility relation</u>. In Kratzer Semantics, it is done by the <u>modal base</u>. The two notions are very similar; since we are talking mostly about the linguistic literature, we'll discuss modal bases next.

3.3 The relativized strict conditional

With the introduction of an accessibility relation R, the truth of the conditional becomes world-dependent.

Definition 2 (Relativized strict conditional)
'*if p then q*' is true at $\langle w, R\rangle$ iff at all worlds w' such that wRw' and p is true at w', q is also true.

- The relativized strict conditional shares most (dis)advantages with the strict conditional, <u>except</u> for the non-contingency of the latter.

Next, we will look at Kratzer's semantics for conditionals. We will do so in two steps: first a simple approach which yields a relativized strict conditional, then a more sophisticated one which does away with more of the problematic validities.

4 Kratzer Semantics[1]

4.1 Preliminaries

Basic idea: Conditionals are modal expressions.

- Modalized sentences come with or without '*if*'-clauses:

(12) a. Joe must be in his office.
b. Mary may leave.

(13) a. If today is Wednesday, Joe must be in his office.
b. If we are finished, Mary may leave.

- If we have an analysis for the sentences in (12), we almost have an analysis for sentences like (13).
- Schematically, we can think of the logical form of these sentences as in (14) and (15): the antecedent modifies the modal operator.

(14) a. MUST(Joe be in his office)
b. MAY(Mary leave)

(15) a. MUST[today is Wednesday](Joe be in his office)
b. MAY[we are finished](Mary leave)

➽ Still needed:

- A general theory of modality (with or without '*if*'-clauses)
- An account of the role of '*if*'-clauses
- A fix for conditionals without (overt) modals:

(16) If today is Wednesday, Joe is in his office.

➽ Modality is about relations of consequence and consistency.

- '*MODAL(p)*' is a statement about the relationship between p (the prejacent) and a certain body of information, call it MB.

[1] Kratzer (1977, 1978, 1979, 1981a,b, 1986, 1989, 1991a,b, 2012)

4.2 Modal language

4.2.1 Some typical modal expressions (in English)

(17) a. Auxiliaries
‘*must, may, might, can, could, shall, should, will, would, have to, ought to*’
b. Adjectives
‘*invincible, permissible, payable, fragile, mortal, edible, fertile*’
c. Adverbs
‘*presumably, possibly, probably, necessarily*’
d. “Inherent modality”
‘*This room seats fifty people; this car goes eighty miles per hour*’
e. Progressive
‘*John was drawing a circle*’ (Dowty, 1977)
f. Temporal conjunctions
‘*Mary defused the bomb before it exploded*’ (Beaver and Condoravdi, 2003)
g. Present tense
‘*John submits his paper*’ (Kaufmann, 2005)
h. Infinitives (in certain constructions)
‘*What to do when your car breaks down*’
‘*The person to call is John*’

4.2.2 Related phenonema

- Propositional attitudes
‘*believe/doubt/wish/suppose (that) p*’

- Evidentiality
‘*That restaurant is supposed to be good; You must be tired!*’

- Verbal mood

(18) a. Hans ist schon da.
Hans is-IND already here
‘Hans is already here.’
b. {Ich dachte / Maria sagte / ...} Hans sei schon da.
I thought Maria said Hans is-SUB here
‘{I thought / Maria said / ...} Hans was already here.’

- Sentential mood

(19) a. You are writing a term paper (declarative)
b. Are you writing a term paper? (interrogative)
c. Write a term paper. (imperative)
d. etc.

4.2.3 Two dimensions of variation

Modal flavor. (Body of information *MB*). This can nicely be illustrated with the various senses of “necessity.” The statement that ‘*p is necessary*’ takes on different meanings for different

Table 1: Some modal flavors

Term	Content	Meaning of “necessity”
Circumstantial	(relevant facts)	‘*p is the case*’
Epistemic	(knowledge)	‘*p is known*’
Doxastic	(beliefs)	‘*p is believed*’
Stereotypical	(normal course of events)	‘*p is normal*’
Deontic	(obligations, laws)	‘*is required to p*’
Volitive	(decisions)	‘*is willing to p*’
Dispositional	(abilities)	‘*cannot but p*’
Buletic/Desiderative	(desires, preferences)	‘*wants p*’
Teleological	(strategies, goals)	‘*aims at p*’
...		

modalities. Table 1 is a collection of examples from the literature.[2] Formally, the modal flavor comes about through the interplay between modal base and ordering source (see below).

Modal force. (Strength of the connection between A and p): Necessity, Possibility, Likelihood, ...

Intuitively, modal force is a matter of degree. However, standard modal logic only deals with necessity and possibility – there are no degrees.

There are some extensions, however, among them Kratzer’s “ordering sources,” which gives us a notions like relative likelihood: p is more (less) likely than q. We will deal with them in detail.

4.3 Conversational backgrounds

Kratzer’s implementation makes crucial use of a particular notion of a conversational background.[3]

Definition 3 (Conversational background)
A conversational background is a function from worlds to sets of propositions.

(i.e., a function $f : W \mapsto \wp(\wp(W))$)

- Kratzer allows for various kinds of conversational backgrounds. This variation contributes to the selection of the modal flavors listed earlier.

a. epistemic conversational background of agent x
$f_x^{epist}(w) = \{p \subseteq W | p \text{ is known to } x \text{ at } w\}$

b. deontic conversational background $f^{deon}(w) = \{p \subseteq W | p \text{ is required at } w\}$

c. bouletic conversational background of agent x
$f_x^{boul}(w) = \{p \subseteq W | p \text{ is desired by } x \text{ at } w\}$

d. ...and so on. The list is open-ended.

[2] Take this with a grain of salt – the terminology is messy in this area.

[3] Kratzer got the term from Stalnaker, but her use of it in the analysis of modality goes beyond Stalnaker’s intended interpretation as a representation of what is taken for granted by the interlocutors.

Table 2: Properties of conversational backgrounds and the induced accessibility relations, along with the corresponding axioms in Modal Logic. Free variables w, w', w'' are universally quantified over.

Conv. Background	*Accessibility Relation*	*Axiom*	
consistency	seriality	$\Box_R\varphi \Rightarrow \Diamond_R\varphi$	(D)
$R_w \neq \emptyset$	$\exists w'.wRw'$		
realism	reflexivity	$\Box_R\varphi \Rightarrow \varphi$	(T)
$w \in R_w$	wRw		
total realism	identity	$\Box_R\varphi \Leftrightarrow \varphi$	
$R_w = \{w\}$	$wRw' \Leftrightarrow w = w'$		
positive introspection	transitivity	$\Box_R\varphi \Rightarrow \Box_R\Box_R\varphi$	(4)
$w' \in R_w \Rightarrow R_{w'} \subseteq R_w$	$wRw' \wedge w'Rw'' \Rightarrow wRw''$		
negative introspection	euclidity	$\Diamond_R\varphi \Rightarrow \Box_R\Diamond_R\varphi$	(5)
$w' \in R_w \Rightarrow R_w \subseteq R_{w'}$	$wRw' \wedge wRw'' \Rightarrow w'Rw''$		

- Relation to modal logic: Each conversational background f corresponds to a unique accessibility relation R^f:[4]

$$wR^f v \text{ iff } v \in \bigcap f(w)$$

- Notes on how to intersect sets of propositions, where W is the set of all worlds (the second is defined by Kratzer):

$$\bigcap\{\emptyset\} = \emptyset; \quad \bigcap\emptyset = W; \quad \bigcap\{W\} = W$$

- Properties of conversational backgrounds: Just as properties of accessibility relations are a central theme in modal logic, we may impose certain formal properties on certain kinds of conversational backgrounds, to go with certain modal flavors. Kratzer mentions first three in Table 2; the other two are important as well; and there are more.
(Kaufmann and Kaufmann, 2015)

Modals are generally interpreted relative to two parameters: a modal base and an ordering source. Both are represented as conversational backgrounds. We introduce them one-by-one.

4.4 One-parameter modality and the relativized strict conditional

Modal bases represent facts, knowledge, or inviolable assumptions.

- Typical examples: epistemic, doxastic, circumstantial
(see below for more on the last of these)

- Formal properties:

  - Modal bases are generally consistent.

[4] The converse is not unique: a given accessibility relation may be induced by more than one conversational background.

– Kratzer (2012) requires that modal bases are always realistic. (This requirement was not present in the earlier version of the paper, Kratzer, 1981a, and it is not universally accepted.)

- The term "modal base" is sometimes used for the conversational background f itself (i.e., a function from worlds to sets of propositions), sometimes for its extension $f(w)$ at world w (i.e., a set of propositions), and sometimes for $\bigcap f(w)$ (i.e., a set of worlds). For the third sense in particular, a separate term would come in handy. I will use the term modal background for it. (Cariani et al., 2013; Kaufmann and Kaufmann, 2015)

- Given the straightforward connection between conversational backgrounds and accessibility relations, we can reproduce the notions of necessity and possibility that are standard in modal logic. Let w, f be a possible world and modal base.

(20) p is a necessity at $\langle w, f\rangle$ iff p is a consequence of $f(w)$ (i.e., $\bigcap f(w) \subseteq p$) iff $\Box p$ is true at $\langle w, R^f\rangle$

(21) p is a possibility at $\langle w, f\rangle$ iff p is consistent with $f(w)$ (i.e., $\bigcap f(w) \cap p \neq \emptyset$) iff $\Diamond p$ is true at $\langle w, R^f\rangle$

4.5 The restrictor analysis

Kratzer's approach to the Logical Form of conditionals (as opposed to the interpretation of modal operators) is known as the restrictor analysis.

This requires a bit of explanation, although it's not a topic I plan to enter in these lectures (unless you ask me to). See Gibbard (1981); Kratzer (1986) for the origins of the debate, and Gillies (2010); Khoo (2013); Kaufmann and Kaufmann (2015), for recent developments.

- Traditionally, the '*if-then*' construction has been thought of in terms of a binary sentential connective; the question has been (and is) how that connective is to be interpreted.

- Kratzer assumes that the main job of '*if*'-clauses is to restrict the quantificational domains of modal (and other) operators. This is not really what sets her approach apart from the traditional view (since one can give a binary sentential operator an interpretation which mimicks this effect of the '*if*'-clause – see Gillies, 2010).

  Rather, the real difference is this: Kratzer does not assume that each '*if*'-clause introduces its own modal operator. These two ingredients work independently. '*If*'-clauses require the presence of an operator, but multiple '*if*'-clause may share a single one.

  In this section we will first see what thismeans, then we'll look at the origins of the idea and some evidence for it.

Definition 4 (Update)
Let f be a conversational background and p a proposition. The result of updating f with p, written $f[p]$, is defined as follows: for all worlds w, $f[p](w) = f(w) \cup \{p\}$.

- Notice that $f[p]$ is again a conversational background.

- In the literature, including Kratzer's writings, you may also see the notation 'f^+' or 'f^{+p}' for the result of updating f with p.
- In the following definition, the consequent q may contain further '*if*'-clauses. All '*if*'-clauses are "scooped up" and used to modify the accessibility relation in the course of the evaluation.

Definition 1
'*MOD[p]q*' is true at $\langle w, f\rangle$ '*MOD q*' is true at $\langle w, f[p]\rangle$.

(22) '*MOD if p, q*' is true at world w relative to f
  a. iff '*MOD q*' is true at w relative to $f[p]$;
  b. iff for {all / some / ...} worlds v such that $wR_{f[p]}v$, q is true.

Notice again the connection to the above definition of relativized strict modality.

Semantically, the cumulative constraints come down to the same thing as an update with their conjunction:

(23)
$$\begin{aligned} f[p][q] &= \lambda w[f[p](w) \cup \{q\}] \\ &= \lambda w[\lambda v[f(v) \cup \{p\}](w) \cup \{q\}] \\ &= \lambda w[[f(w) \cup \{p\}] \cup \{q\}] \\ &= \lambda w[f(w) \cup [\{p\} \cup \{q\}]] \\ &= \lambda w[f(w) \cup \{p, q\}] \end{aligned}$$

So under the restrictor approach, the Import-Export principle falls out for free:

'*if p, then (if q, r)*' $\Leftrightarrow$ '*if p and q, then r*'

4.6 More on the restrictor analysis

- Interactions with quantifiers
- The Ramsey Test

4.6.1 Interactions with quantifiers.

What do the following sentences mean?[5]

(24)
  a. Sometimes, if a girl gets a chance, she bungee-jumps.
  b. Never, if a girl gets a chance, does she bungee-jump.
  c. Usually, if a girl gets a chance, she bungee-jumps.

- Intuitively, these sentences mean that some girls, no girls, or many girls who get a chance bungee-jump. But this is hard to explain if '*if*' is a (material or strict conditional) operator that is interpreted separately from the adverb ('*usually*', '*never*', '*sometimes*').

  (25) a. {Sometimes / Never / Usually} the conditional "if a girl gets a chance, she bungee-jumps" is true.

[5]The examples are taken from von Fintel's (1998) discussion of Lewis (1975).

b. For {some / no / most} girls x, the conditional "if x gets a chance, x bungee-jumps" is true.

– Material conditional:

For {some / no / most} girls x, either x does not get a chance or x bungee-jumps.

This is false for '*some*' and '*most*' and true for '*no*' because most girls don't get a chance. But that's not what the sentences mean.

– Strict conditional:

For {some / no / most} girls x, x bungee-jumps at all worlds at which x gets a chance.

This is also not what the conditionals say. For instance, in order for the '*some*'-variant to be true, there need not be any girl who <u>necessarily</u> bungee-jumps if she gets a chance.

➽ The sentences mean something like this:

(26) {Some / No / Most} girls <u>who get a chance</u> bungee-jump.

- Lewis (1975) took facts like this to show that '*if*'-clauses can, at least in some cases, be used to restrict quantifier domains. Such examples, he claimed, do not contain an independent conditional sentential connective at all.

> The <u>if</u> of our restrictive <u>if</u>-clauses should not be regarded as a sentential connective. It has no meaning apart from the adverb it restricts. The <u>if</u> in <u>always if. . . ,. . .</u>, <u>sometimes if. . . ,. . .</u>, and the rest is on a par with the nonconnective <u>and</u> in <u>between. . . and. . .</u>, with the nonconnective <u>or</u> in <u>whether . . . or . . .</u>, or with the nonconnective <u>if</u> in <u>the probability that . . . if . . .</u>. It serves merely to mark an argument place in a polyadic construction.

- Kratzer adopts this proposal and extends it in two important directions:

  – According to Kratzer, **all** '*if*'-clauses restrict quantifier domains.
  In ordinary conditionals, the '*if*'-clause restricts the domain of a modal operator.

  – Even conditionals with no overt modal contain a **covert** modal operator that is restricted by the '*if*'-clause.

4.6.2 The Ramsey Test

- Ramsey (1929) relates the interpretation of (indicative) conditionals to the dynamics of belief change:

(27) If two people are arguing 'If p will q?' and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q . . . We can say they are fixing their degrees of belief in q given p.

- Suppose a person's "stock of knowledge" is represented as an epistemic modal base f. Then the "addition of p" can be modeled simply by adding updating f with p, as shown above.

4.7 Two-parameter modality and the (relativized) variably strict conditional

Ordering sources represent defeasible preferences, desires, or normalcy assumptions. They can be inconsistent (both internally and with the modal base).

Basic idea: To evaluate '*MODAL(p)*', check whether p is a consequence of / consistent with all ways of enriching the modal base with propositions from the ordering source, maintaining consistency.

There are (at least) wwo ways of spelling this out formally: Premise Semantics and Ordering Semantics. For now we'll focus on the latter; the former is described in Appendix D.

4.8 Ordering Semantics

Basic idea: Rank worlds according to the relevant criterion (e.g., goodness, likelihood); check whether p is true at all / some of the "best" ones.[6]

- Induced order on the set of worlds given $g(w)$ (the value of the ordering source at w, a set of propositions): u is at least as good as v iff all the propositions in $g(w)$ that are true at v are also true at u (and possibly more).

  (28) $u \leqslant_{g(w)} v$ iff $\{p \in g(w) | v \in p\} \subseteq \{p \in g(w) | u \in p\}$

  Notes:

  - $\leqslant$ is naturally read "less than or equal," but here $u \leqslant_{g(w)} v$ means that u verifies at least all the propositions in $g(w)$ that v verifies (and possibly more). Some people find this a bit counterintuitive; some even reverse the symbol and use '$\geqslant$' for the same notion. I stick with Kratzer's notation; but watch out when you read other things.
  - This order does not give us "degrees" of conformity with the ordering source. We only get a way of comparing worlds and telling which is better/worse; there is no absolute measure of "goodness" associated with the worlds.
  - The order is defined on all worlds; however, for the purposes of interpreting modals, we are only interested in its restriction to the modal-base worlds.
  - For any g, w, the induced order is a pre-order. That is, it is
    * reflexive: $v \leqslant_{g(w)} v$ for all v;
    * transitive: if $r \leqslant_{g(w)} u \leqslant_{g(w)} v$ then $r \leqslant_{g(w)} v$ for all r, u, v

- **Limit Assumption:** Every modal-base world either is, or is outranked by, a modal-base world that is minimal under the induced order.

  Formally: For all $v \in \bigcap f(w)$, there is a $u \in \bigcap f(w)$ such that (i) $u \leqslant_{g(w)} v$ and (ii) for all $r \in \bigcap f(w)$, if $r \leqslant_{g(w)} u$ then $u \leqslant_{g(w)} r$.

  Let $\mathbf{O}(w, f, g)$ be the set of minimal worlds in $\bigcap f(w)$ under $\leqslant_{g(w)}$. Then necessity and possibility can be defined as follows:

[6] This is akin to Lewis's (1973) theory of counterfactuals. Most of Kratzer's work is formulated in this framework. See Lewis (1981) for a comparison of Premise Semantics and Ordering Semantics (see also Kaufmann, 2017).

(29) p is an <u>OSLA-necessity</u> at $\langle w, f, g\rangle$ iff $\mathbf{O}(w, f, g) \subseteq p$

(30) p is an <u>OSLA-possibility</u> at $\langle w, f, g\rangle$ iff $\mathbf{O}(w, f, g) \cap p \neq \emptyset$

- **General Case:**

  (31) p is an <u>OS-necessity</u> at $\langle w, f, g\rangle$ iff for all $v \in \bigcap f(w)$ there is a $u \in \bigcap f(w)$ such that
    a. $u \leqslant_{g(w)} v$ and
    b. for all $r \in \bigcap f(w)$, if $r \leqslant_{g(w)} u$ then $r \in p$.

  - In words: For all worlds v in the modal base, there is a world u in the modal base that is at least as close to the ideal and that is not equalled or outranked by any world r in the modal base in which p is false.
  - In still other words: As you "hop across" modal-base worlds closer and closer to the ideal, you eventually enter p-territory and won't leave it again.

  (32) p is an <u>OS-possibility</u> at $\langle w, f, g\rangle$ iff there is some $v \in \bigcap f(w)$ such that for all $u \in \bigcap f(w)$, if $u \leqslant_{g(w)} v$ then there is some $r \in \bigcap f(w)$ such that $r \leqslant_{g(w)} u$ and $r \in p$.

<u>Note:</u> Pre-orders over sets of possible worlds are also quite popular in Artificial Intelligence (less so their derivation from ordering sources). See in particular the discussion on "relative likelihood" in Halpern (2003) and its relationship to other representations of uncertainty, such as probability and possibility. See also Veltman (1996) for a similar approach to the belief update with defaults.

In the rest of this handout, I will mainly stick to Ordering Semantics. But we will see more Premise Semantics later in these lectures.

5 Conditionals

5.1 Indicatives and Strengthening the Antecedent

- The relativized strict analysis accounts nicely for the context-dependence of conditionals. A given conditional can be simultaneously true with respect to one accessibility relation or modal base and false with respect to another. Thus (33) may be objectively (circumstantially) true, but believed to be false by a speaker with insufficient information or false beliefs.

  (33) If this material is heated to 500°C, it will burn.

- However, the definition so far, like the material conditional and the strict conditional, fails to account for the invalidity of the problematic inference patterns. We focus on Strengthening of the Antecedent; the others are left as an exercise. Notice that (33) can be true while (34) is false.

  (34) If this material is placed in a vacuum chamber and heated to 500°C, it will burn.

- Kratzer's solution: Like all modals, '*will*' in (33) and (34) is interpreted relative to an ordering source (in this case, a "stereotypical" one). Making the Limit Assumption, this means that only the most typical ways for the material to be heated are relevant for the truth or falsehood of the conditional.

- This offers a solution to the above problem. Suppose the material is normally not placed in a vacuum chamber. Then every antecedent-world at which it is, is outranked in normalcy by one at which it is not, thus (33) may be true while (34) is false. Technically, the conditional (35) is a (weak) necessity.

(35) If this material is heated to 500°C, it won't be in a vacuum chamber.

- So in effect, <u>while the modal base and the ordering source are the same in both (33) and (34)</u>, the truth of the conditional depends on different sets of worlds.

a. (33): Worlds at which the material is heated and not in a vacuum.
b. (34): Worlds at which the material is heated and in a vacuum.

- The other problematic inference patterns are also invalidated. Exercise: show this for Transitivity (Hypothetical Syllogism).

5.2 Counterfactuals

- Unlike indicative conditionals, counterfactuals are typically used when the antecedent is in doubt or known to be false. (There are a few exceptions, but we can ignore them.)

  In such a case, the antecedent cannot be added to the modal base consistently.

- Kratzer: "A counterfactual is characterized by an empty modal base f and a totally realistic ordering source g."
- Recall that $\cap\emptyset = W$. Thus the worlds in the modal base are <u>all</u> the worlds in W.
- A totally realistic ordering source means that $\cap g(w) = \{w\}$. This ensures that w is a minimal element in the order induced by $g(w)$ (i.e., w is closest to itself): For each $w' \neq w$, there must be some proposition p in $g(w)$ such that $w \in p$ and $w' \notin p$. But since g is realistic, there is no $q \in g(w)$ such that $w' \in q$ and $w \notin q$.
- The invalid inferences discussed above for indicative conditionals (Strengthening of the Antecedent etc.) are invalid for counterfactuals as well. Consider the famous example due to Lewis (1973):

(36) a. If kangaroos had no tails, they would topple over.
b. If kangaroos had no tails and walked on crutches, they would topple over.

The fact that (36a) does not entail (36b) is taken care of in the same way as (33) and (34) above.

- <u>Note:</u> This treatment of counterfactuals is closely related to the "ordering semantics" of Lewis (1973). For a comparison, see Lewis (1981). These issues are largely beyond the scope of this class (unless there is interest).
- This is all we will say about counterfactuals at this time. I should note that I do not actually endorse Kratzer's analysis (although an analysis in a variant of her framework is possible). There will be more later in these lectures.

6 Conditionals with non-epistemic modals in the consequent

- Kratzer assumes that a <u>covert</u> modal operator is present in case there is no overt modal for the '*if*'-clause to restrict.

  In the original versions of her writings, Kratzer assumed (or was generally taken to assume) that this silent operator is only present when there is no overt modal (perhaps as a kind of "last resort"); but where there is an overt modal, its modal base is restricted by the '*if*'-clause.

  This was the analysis for all modals, epistemic or otherwise. It is still on display (for deontic modals) in Chapter 2 of Kratzer (2012): see her paraphrase of the truth conditions of her (60) and (61) on page 67 (in the prose in the lower half of the page).

- But subsequent work (starting in the the mid-90's, after the publication of the original versions of Kratzer's papers) cast doubt on this assumption: covert modals seem to coexist (at least in some cases) with overt modals; in particular, they seem to be <u>always</u> present with <u>non-epistemic</u> (e.g., deontic or ability) modals.

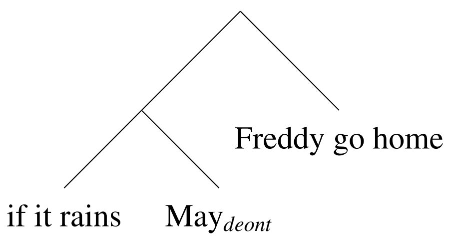
  In Chapter 4 of Kratzer (2012), Kratzer acknowledges that there cases where the overt modal in the consequent is not restricted by the '*if*'-clause.

- Her framework has no trouble accommodating this possibility: we just have to assume that covert modals can coexist with overt ones. Consider (37).

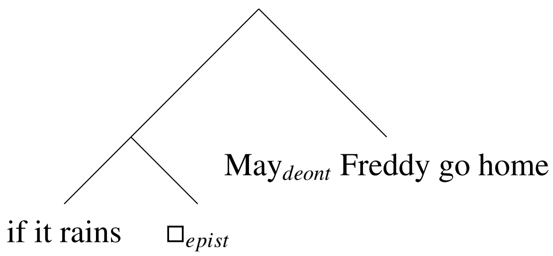
  (37) a. Freddy may go home.
  b. If it rains, Freddy may go home.

  Once we admit the possibility of covert operators in conditionals, what is the structure for (37b)? In (38a), the deontic modal in the consequent is modified by the '*if*'-clause; in (38b), the '*if*'-clause modifies a covert epistemic modal, which takes the overt deontic modal in its scope.

  (38) a.



  b.



  Schwager (2006a,b) and Kaufmann and Schwager (2009) dub these two structures the <u>Overt Conditional Operator (OCO)</u> and <u>Covert Conditional Operator (CCO)</u> construal, respectively. The distinction was discussed earlier, however.

- This section reviews some of the arguments that established that non-epistemic modals are not restricted by '*if*'-clauses.

So assume (until further notice) the original version of the Kratzerian account of conditionals: (i) if there is a modal, it is restricted by the antecedent; thus (ii) the modal base f and ordering source g for the whole conditional are the same as the one for the consequent.[7]

Thus for instance, if the consequent has a deontic modal, then the whole conditional is interpreted with respect to the same deontic modal base (modulo the addition of the antecedent).

The main problem. Frank (1996) and Zvolenszky (2002) point out the following problem with Kratzer's original account:

Fact 1 If A entails B, under Kratzer's analysis, a conditional '*If A, must B*' is necessarily true.

Fact 2 Likewise for the conditional '*If A, may B*', as long as there are A-worlds in the modal base.

- It is not hard to see why this is so: Whatever the modal base is (epistemic, circumstantial, deontic, etc.), after adding the antecedent to it, we end up with a set of worlds ($\bigcap f^{+A}(w)$) throughout which the antecedent is true. But if the antecedent entails the consequent, then the consequent is also true throughout $\bigcap f^{+A}(w)$. Furthermore, even if there is a non-trivial ordering source, the truth of the conditional still depends only on worlds in the modal base.
- But clearly, deontic conditionals of this form can be false. The best counterexamples are ones with root modals in which A and B are the same. Here are some examples from Zvolenzsky:

(39) a. If teenagers drink, then teenagers must drink.
b. If teenagers drink, then teenagers may drink.
c. If I file my taxes, then I want to file my taxes.
d. If children don't eat spinach, then children shouldn't eat spinach.

- All of these sentences are predicted true under Kratzer's account with respect to <u>any</u> moda base f and ordering source g.
- Another one of Zvolenzsky's examples: In fact, Britney Spears does (or did, at the time) have a contract with Pepsi that stipulated that (40a) is true: Among all the cola-drinking worlds, the Pepsi worlds are closer to the ideal than others.

(40) a. If Britney drinks cola in public, she must drink Pepsi.
b. If Britney drinks Coke in public, she must drink Coke.

- Under Kratzer's analysis, (40a) may well be true, but (40b) is necessarily <u>also</u> true. In fact, though, they seem contradictory.

Further problems. We may assume that the indicative in (41a) is interpreted relative to a circumstantial modal base and a deontic ordering source.

(41) a. If Max buys this car, he must pay taxes for it.
b. If Max had bought a car, he would have to pay taxes for it.

[7] There is in fact a subtle difference: Both Kratzer (1981a) and Kratzer (1991b) claim that the consequent is evaluated with respect to modal base f^{+A}, where A is the antecedent, and ordering source g. However, whereas Kratzer (1981a) says that f is the modal base of the <u>antecedent</u> and somehow inherited by the consequent (this is still assumed in Chapter 2 of Kratzer, 2012), in Kratzer (1991b) it is the modal base of <u>the whole conditional</u>.

c. If Max really loved his dog, he should take it for a walk.

- Frank (1996) points out that under Kratzer's analysis, a similar interpretation is not available for counterfactuals like (41b,c). Kratzer states counterfactuals generally are interpreted with respect to an empty modal base and a circumstantial ordering source. Thus $\bigcap f(w) = W$ and $\bigcap g(w) = \{w\}$.
- But then we are in danger of losing the deontic flavor of the conditional: The worlds in the modal base are ranked with respect to their similarity to the actual world, which is not (necessarily) the deontic ideal.
- Frank briefly considers the possibility of operating with two ordering sources (one realistic, one deontic) on the same modal base, but cannot think of a principled way (and neither can I) of specifying how both would interact in order to generate the order.
- So should we just say that counterfactuals can be interpreted with respect to a deontic ordering source (combined perhaps with the empty background f)?

  No. The deontic modal base would be one based on the laws in this world (i.e., the world of evaluation). But what if the antecedent itself implies that the laws are different from the actual ones?

  (42) If Luther hadn't brought about the Reformation, we would still have to pay indulgence.

Yet more problems. This argument is due to Kaufmann and Schwager (2009); there it was presented as a problem for conditionals with imperative consequents; but the same issues arises with (other) modals in the consequent.

(43) a. If you lose your job, you should take a lower-paying one.
b. But if you lose your job and have a comparable offer, you should not take a lower-paying one.

I quote:

> Formally, [(43)] is analogous to standard counterexamples to Strengthening of the Antecedent, the inference from '*if A, C*' to '*if AB, C*'. In Kratzer-style semantics, the invalidity of this pattern is accounted for by allowing that the set of *AB*-worlds relevant for the evaluation of the latter is not contained in the set of *A*-worlds relevant for the former. ...the selection of the relevant worlds is driven by the ordering source. With a bouletic ordering source, Strengthening of the Antecedent fails if *AB* is not the most preferred way for *A* to come about and *C* is true at the most preferred *A*-worlds but false at some or all of the most preferred *AB*-worlds.
>
> In fact, [(43)] is a particularly strong counterexample in that [the prejacents in the consequents] are not only contradictories but contraries of each other. In this case, the set of relevant *AB*-worlds must be disjoint from the set of relevant *A*-worlds in order for both sentences to be true. Thus in [(43)] it must be the case that among all the worlds at which the addressee is laid off, some of the ones at which she does not have a comparable offer are strictly preferred over all the ones at wich she does. But clearly this prediction is wrong: [(43a,b)] do not jointly imply that the speaker wishes that the addressee have no comparable offer in case she loses her job.

The solution (in rough outline). It seems better to choose the antecedent-worlds w' first (for instance, *via* an epistemic modal base) and then evaluate the consequent with respect to the laws at w' (not w). Formally, this means that the modal in the consequent has its own modal base (e.g., circumstantial) and ordering source (e.g., deontic).

- That is Frank's proposal. "Deontic" conditionals are really epistemic ones: The antecedent restricts an epistemic modal base; the deontic modal in the consequent does not take scope over the whole conditional.

- For additional evidence, Frank reminds us that an epistemic modal can appear in these sentences (44c,d), and that it is generally assumed that such an epistemic modal is present (overtly or covertly) in all conditionals (cf. Kratzer, 1991b,a).

  Thus, Frank asks, why not just say that there is an implicit epistemic modal in (44a,b)?

(44) If Max stays with Grandma, ...
   a. he is allowed to take the dog for a walk.
   b. he must take the dog for a walk.
   c. he might be allowed to take the dog for a walk.
   d. he might have to take the dog for a walk.

- Frank's proposal for (44a–d) still runs into some problems. The one for (45) is better (though still not perfect):[8]

(45) a. If Max bought a car, he would have to pay taxes.
   b. $necessary^{g(w)}_{f^{+p}(w)}(necessary^{g'(w')}_{f'(w')}q)$
   c. "For all worlds w' in the (epistemic) modal base in which Max buys a car, the following is true: He pays taxes in all worlds $w'' \in f'(w')$ that are closest to the deontic ideal in w'."

➽ To summarize, I quote from Frank:

> In sum, these observations lead us to the conclusion htat there are in fact no truely deontically modalized if-conditionals. Instead we assume conditionals with a deontic modal operator in the consequent clause to be analyzed in terms of an implicit or explicit epistemically (or circumstantially) based modal operator. The deontic modal adverb is then to be analyzed within the scope of the 'higher' epistemic modal operator.

A Truth functionality

Edgington (1986) presented the following argument that very minimal assumptions ensure that a truth-functional conditional must be the material conditional.

Proposition 1 (Edgington, 1986)
Assume that

[8]There is some residual discussion over the nature of f'. For our purposes, we can assume that it is circumstantial, allowing for some "magic" in ensuring that it is indeed restricted to all the relevant worlds. Frank ultimately favors a somewhat different solution, but the details are very intricate.

(Ea) $\textsc{if}(\ ,\ )$ denotes a truth function, call it $\mathcal{F}_{\textsc{if}}$;

(Eb) sentences of the form $\textsc{if}(p \text{ and } q\,,\ p)$ are tautologous;

(Ec) conditionals can be false.

Then $\mathcal{F}_{\textsc{if}}$ is the truth function of the material conditional.

Proof. Let's assume that '*and*' is $\wedge$. By Assumption (Eb), the sentence $\textsc{if}(p \wedge q\,,\ p)$ is a tautology, i.e., $\mathcal{F}_{\textsc{if}}(p \wedge q, p) \equiv 1$. There are four cases:

	p	q	$p \wedge q$	$\mathcal{F}_{\textsc{if}}(p \wedge q, p) = 1 = \ldots$
(a.)	1	1	1	$\mathcal{F}_{\textsc{if}}(1, 1)$.
(b.)	1	0	0	$\mathcal{F}_{\textsc{if}}(0, 1)$.
(c.)	0	1	0	$\mathcal{F}_{\textsc{if}}(0, 0)$.
(d.)	0	0	0	$\mathcal{F}_{\textsc{if}}(0, 0)$.

Cases (a.–d.) exhaust three of the four possible combinations of arguments of $\mathcal{F}_{\textsc{if}}$. By Assumption (Ec), conditionals can be false, hence $\mathcal{F}_{\textsc{if}}(1, 0) = 0$.

□

B Modal logic

Definition 2 (Languages)
Let $\mathcal{A} = \{p, q, r, \ldots\}$ be a set of propositional variables.

- The language $\mathcal{L}^0_{\mathcal{A}}$ of propositional logic is the smallest set containing $\mathcal{A}$ and such that for all $\phi, \psi \in \mathcal{L}^0_{\mathcal{A}}$, $(\neg\phi), (\phi \wedge \psi), (\phi \vee \psi), (\phi \rightarrow \psi) \in \mathcal{L}^0_{\mathcal{A}}$.
- The language $\mathcal{L}_{\mathcal{A}}$ of propositional modal logic is the smallest set containing $\mathcal{L}^0_{\mathcal{A}}$ and such that for all $\phi \in \mathcal{L}_{\mathcal{A}}$, $(\Box\varphi), (\Diamond\varphi) \in \mathcal{L}_{\mathcal{A}}$.

(46) a. $\Box p \rightarrow p$
b. If p is necessary then p is true.

The semantics of modal logic relies on the notion of possible worlds. This notion is familiar and widely used, although there is (probably) some variation in the metaphysical views people bring to this work. (E.g., Is the world we actually inhabit one of the possible ones? Are the other ones like this one? Which worlds are there, and how do they differ from each other?) For our purposes, we can just assume that possible worlds are something to be defined, not discovered: they are whatever you say they are. And their sole purpose is to fix the truth values of all sentences of the language.

The second ingredient is that of an accessibility relation between possible worlds. What this is depends on the modality in question. This is easiest to make sense of by thinking of some concrete examples. For instance, if we are interested in epistemic modality, '$\Box\varphi$' means "φ is known." Then for each world w, the set of worlds accessible from w is the set of just those worlds that are not ruled out by the agent's knowledge – i.e., those not ruled out by his/her (partial) information. If, again, we want to use deontic modality, then the worlds accessible from w are just those which are consistent with what the law prescribes. And so on.

Definition 3 (Model for $\mathcal{L}_{\mathcal{A}}$)
A model for the interpretation of $\mathcal{L}_{\mathcal{A}}$ is a triple $\mathrm{M} = \langle W, R, V\rangle$, where W is a non-empty set of possible

worlds, $R \subseteq W \times W$ is an accessibility relation, and $V : \mathcal{L}_{\mathcal{A}} \mapsto \wp(W)$ is a function from sentences of $\mathcal{L}_{\mathcal{A}}$ to sets of possible worlds, subject to the following constraints:

$$\begin{aligned}
V(\neg\phi) &= W \setminus V(\phi) \\
V(\phi \wedge \psi) &= V(\phi) \cap V(\psi) \\
V(\phi \vee \psi) &= V(\phi) \cup V(\psi) \\
V(\phi \to \psi) &= W \setminus (V(\phi) \setminus V(\psi)) \\
V(\Box\phi) &= \{w \mid \text{for all } v, \text{ if } wRv \text{ then } v \in V(\phi)\} \\
V(\Diamond\phi) &= \{w \mid \text{for some } v, wRv \text{ and } v \in V(\phi)\}
\end{aligned}$$

Systems: axioms and frame properties We won't be much concerned with topics like axiomatic systems and proof theory. Those topics are what logicians focus on much of the time. But linguists should at least have some working knowledge of some crucial logical notions and methods, since they turn out to have important uses and ramifications in semantic theories.

Classical (non-modal) propositional logic can be axiomatized in various ways; a fairly common system is characterized by the three schemata in (Ł) (named after the Polish logician Jan Łukasiewicz), together with (MP), the rule of modus ponens.

(Ł) a. $\phi \to (\psi \to \phi)$
b. $(\phi \to (\psi \to \theta)) \to ((\phi \to \psi) \to (\phi \to \theta))$
c. $(\neg\phi \to \neg\psi) \to ((\neg\phi \to \psi) \to \phi)$

(MP) $\dfrac{\phi, \phi \to \psi}{\psi}$

The modal logics we are interested in typically include all the axioms of propositional logics, plus the schema (K) (named after the American philosopher Saul Kripke) and the rule (N) of necessitation.

(K) $\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$

(N) $\dfrac{\phi}{\Box\phi}$

All of these things are generall taken for granted in linguistics (and built into the standard possible-worlds models linguists use). On top of that, however, there are some axiom schemata that do lead to non-trivial empirical predictions, and that one may or not want to include in one's semantic representation of a given linguistic expression. Some of them (there are more) are listed here, along with the labels they usually receive in the literature:

(T) $\Box\phi \to \phi$
(D) $\Box\phi \to \Diamond\phi$
(4) $\Box\phi \to \Box\Box\phi$
(5) $\Diamond\phi \to \Box\Diamond\phi$

What these axioms express depends on what the necessity operator is intended to model: knowledge, belief, obligation, metaphysical necessity, etc. Try some of these interpretations and decide for yourself how plausible each of the axioms is for them.

The axioms are not independent: for instance, (T) implies (D), and (T) and (5) jointly imply (4).

As an alternative to the axiomatic method, the underlying properties of various modalities can also

Table 3: Common systems of modal logic

Name	*Axioms*
T	**K** + (T)
S4	**K** + (T) + (4)
S5	**K** + (T) + (5)
KD45	**K** + (D) + (4) + (5)

be expressed in terms of constraints on the models, specifically in terms of <u>properties of accessibility relations</u>. Some terminology helps in spelling this out.

We said above that a model is a structure $M = \langle W, R, V\rangle$. The <u>frame</u> of this model is the pair $\langle W, R\rangle$ – i.e., it specifies the accessibility relation but not the valuation function. The idea is this: Certain properties of $\langle W, R\rangle$ ensure that certain logical relations between modal sentences hold no matter how the language in question is interpreted (i.e., regardless of V).

Definition 4 (Truth and validity)
A sentence ϕ is

a. true at a world w in a model $\langle W, R, V\rangle$ iff $w \in V(\phi)$.

b. true in a model $\langle W, R, V\rangle$ iff $W \subseteq V(\phi)$

c. valid on a frame $\langle W, R\rangle$ iff ϕ is true in all models based on $\langle W, R\rangle$.

With these notions at hand, we can establish <u>correspondences</u> between axioms and frame properties. These are results to the effect that a given axiom (or all instances of a given axiom schema) are valid on a frame $\langle W, R\rangle$ if and only if R has a certain property. Here's a simple example:

Proposition 2
$\Box\phi \rightarrow \phi$ is valid on a frame $\langle W, R\rangle$ iff R is reflexive.

Proof. ($\Rightarrow$) Suppose R is not reflexive. Thus there is a world $w \in W$ such that $\neg wRw$. Let V be such that (i) for all worlds v, if wRv then $v \in V(\phi)$; and (ii) $w \notin V(\phi)$. By (i), $w \in V(\Box\phi)$; by (ii), $w \notin V(\phi)$. Thus the formula is not valid on $\langle W, R\rangle$.

($\Leftarrow$) Suppose R is reflexive and the formula is not valid. Thus there is a world $w \in W$ such that (i) $w \in V(\Box\phi)$ and (ii) $w \notin V(\phi)$. By (i) and the reflexivity of R, $w \in V(\phi)$, contradicting (ii). □

Here are some more correspondences between axiom schemata that are very commonly encountered in the literature on knowledge, belief and common ground (e.g., Stalnaker, 2002), and frame properties. It's a very useful exercise to prove (some of) these; but I don't expect to spend much time on this unless there is interest.

(47) a. Axiom (D) is valid on $\langle W, R\rangle$ iff R is <u>serial</u> [consistency]
(i.e., for all worlds w, there is some world v such that wRv).
b. Axiom (4) is valid on $\langle W, R\rangle$ iff R is <u>transitive</u> [positive introspection]
(i.e., for all worlds w, v, u, if wRv and vRu, then wRu).
c. Axiom (5) is valid on $\langle W, R\rangle$ iff R is <u>euclidian</u> [negative introspection]
(i.e., for all worlds w, v, u, if wRv and wRu, then vRu).

Figure 1 is helpful in clarifying the import of transitivity and euclidity.

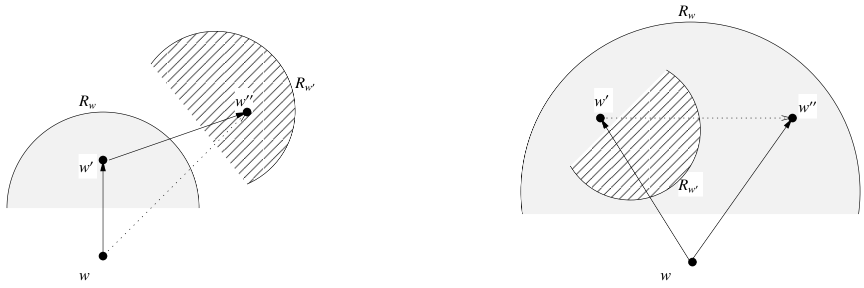


Figure 1: Violations of transitivity (left) and euclidity (right).

C "Circumstantial" modality

- The notion of "circumstantial" modality was introduced by Kratzer (1981a). The difference between epistemic and circumstantial modal bases can be subtle and has at times led to puzzlement.

- Here is an example from Kratzer (1991b) (see Kratzer, 2012, p. 52 for a similar German example):

  (48) a. Hydrangeas can grow here. (circumstantial)
  b. There might be hydrangeas growing here. (epistemic)

  The two sentences differ in meaning in a way which is illustrated by the following scenario.

  – Suppose I acquire a piece of land in a far away country and discover that soil and climate are very much like at home, where hydrangeas prosper everywhere. Since hydrangeas are my favorite plants, I wonder whether they would grow in this place and inquire about it. The answer is (48a). In such a situation, the proposition expressed by (48a) is true. It is true regardless of whether it is or isn't likely that there are already hydrangeas in the country we are considering. All that matters is climate, soil, the special properties of hydrangeas, and the like.
  – Suppose now that the country we are in has never had any contacts whatsoever with Asia or America, and the vegetation is altogether different from ours. Given this evidence, my utterance of (48b) would express a false proposition. What counts here is the complete evidence available. And this evidence is not compatible with the existence of hydrangeas.

- Kratzer (1991b) further writes:

  > Epistemic modality is the modality of curious people like historians, detectives, and futurologists. Circumstantial modality is the modality of rational agents like gardeners, architects and engineers. A historian asks what might have been the case, given all the available facts. An engineer asks what can be done given certain relevant facts.

- Kratzer (2012) relates the distinction that but between "epistemic" and "root" modality (p. 50), which is also somewhat ill-undertood. At the same time she admits that "[i]n the end, those terms are all likely to be problematic in some way or other. . . " We should read more about it.

D Premise Semantics

Basic idea: Put together all the facts and preferences; check whether p follows from / is compatible with the result.[9]

- A premise set is a consistent set of propositions.
- Relative to $\langle w, f, g\rangle$, a premise set is a consistent set containing $f(w)$ and some subset of $g(w)$.
- The set of all premise sets at $\langle w, f, g\rangle$ is

  (49) $\text{Prem}(w, f, g) = \{f(w) \cup X \mid X \subseteq g(w) \text{ and } f(w) \cup X \text{ is consistent}\}$

- **Limit Assumption:** Every premise set is contained in a maximal premise set.

  Formally: For all $X \in \text{Prem}(w, f, g)$ there is a $Y \in \text{Prem}(w, f, g)$ such that (i) $X \subseteq Y$ and (ii) for all $Z \in \text{Prem}(w, f, g)$, if $Y \subseteq Z$ then $Y = Z$.

  If we make the Limit Assumption, necessity and possibility can be defined as follows.

  (50) p is a PSLA-necessity at $\langle w, f, g\rangle$ iff p is a consequence of all maximal premise sets in $\text{Prem}(w, f, g)$.

  (51) p is a PSLA-possibility at $\langle w, f, g\rangle$ iff p is consistent with all maximal premise sets in $\text{Prem}(w, f, g)$.

  But the Limit Assumption may not always hold. For instance, there may be infinite increasing chains of premise sets, where each one is strictly contained in a larger one.

- **General Case:** If we cannot (or will not) make the Limit Assumption, the definitions of necessity and possibility have to be a bit more complicated:

  (52) p is a PS-necessity at $\langle w, f, g\rangle$ iff every set X in $\text{Prem}(w, f, g)$ has a superset in $\text{Prem}(w, f, g)$ of which p is a consequence.

  (53) p is a PS-possibility at $\langle w, f, g\rangle$ iff there is an X in $\text{Prem}(w, f, g)$ such that p is consistent with all of its supersets in $\text{Prem}(w, f, g)$.

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[9]This is akin to Goodman's (1947) theory of counterfactuals; indeed, some of Kratzer's work on counterfactuals can be seen as elaborations on Goodman (e.g., Kratzer 1989).

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