The psychology of conditionals
Class 3
Ramsey and de Finetti

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Today's class

Traditional research on truth tables established a result that came to be known as *the defective truth table*, in which *not-p* outcomes are judged to be "irrelevant" to *if p then q*.

There is a relation between the defective truth table and the position of Ramsey and de Finetti that:

P(if p then q) = P(q|p)

If this identity holds, then a Bayesian account can be given of people's reasoning for the purposes of belief revision and updating.

Recall the material conditional truth table

If you buy this lottery ticket (p), you will win lots of money (q)

1 = true, 0 = false

p \ *q*	1	0
1	1	0
0	1	1

Wason (1966) and the "defective" table

"If a card has a vowel on one side, it has an even number on the other side."

"Subjects assume implicitly that a conditional statement has, not two truth values, but three: true, false, and 'irrelevant'."

Note that the above is a <u>general</u> conditional and not a <u>singular</u> conditional. Also Wason speaks of "irrelevant" as both a truth value of the conditional and a property of some of the cards. But in this first work, he does not use the word "defective" for the table.

The so-called "defective" truth table

If you buy this lottery ticket (p), you will win lots of money (q)

1 = true, 0 = false, Irrelevant

p \ *q*	1	0
1	1	0
0	Irrelevant	Irrelevant

Ambiguities and uncertainties

There are a number of ambiguities and uncertainties in the literature on the "defective" truth table. We have seen two of them right at the beginning of the literature on it.

Is the conditional singular or general?

Does *irrelevant* refer to a third truth value and thus to a semantic property of a conditional, or does it refer to a pragmatic or semantic property of the objects referred to by the conditional?

First ambiguity

Evans et al. (2003) pointed out experiments on conditionals should clearly distinguish between singular and general conditionals. We asked about a given singular conditional, "How likely is the following claim to be true (false) of a card drawn at random?"

"If the card is yellow then it has a circle printed on it."

A general conditional, "If a (or any) card is yellow …", does not have some third, "irrelevant", truth value when a card drawn at random turns out to be red. See Cruz & Oberauer (2014) on general conditionals.

Second ambiguity

Suppose a card is to be drawn at random and "the card" in the following refers to it:

If the card is yellow then it has a circle printed on it.

Many objects are "irrelevant" to this conditional in the ordinary sense.

But now suppose that the selected card is red. Does that imply this conditional is neither true nor false, but has a third value of some type?

Kneale & Kneale (1962)

Kneale & Kneale compare a range of conditional speech acts: Conditional predictions, conditional promises, and conditional orders. They were the first to use "defective" for false antecedent cases.

In false antecedent cases, the prediction is not verified, the promise is not kept, and the order is not obeyed. The conditional utterances have no application is such cases.

The truth table is said to be "defective". No truth value at all, and not some third truth value, is assigned in false antecedent cases. The table is displayed.

Wason & Johnson-Laird (1972)

Kneale & Kneale do not base their account on Quine, but Wason & Johnson-Laird do, concluding, "Hence, as Quine (1952) remarked, the ordinary conditional is a conditional assertion rather than the assertion of the conditional … On this presuppositional analysis, the conditional has an incomplete truth table: no value is specified for those cases where antecedent is false … "

The "defective" truth table then follows, with "void" used as the third "value".

Conditional bets

Ramsey (1926, 1929) and de Finetti (1936, 1937) compared conditional assertions to conditional bets, like:

I bet you £10 that, if the card is yellow, then it has a circle printed on it.

The bet is won, and the conditional assertion true, when the selected card is yellow and has a circle on it. It is lost, and the assertion is false, when the card is yellow and does not have a circle on it. The conditional bet is "void", and so is the conditional assertion, when the card is not yellow (Politzer, Over, & Baratgin, 2010).

What does "void" mean?

It is sometimes said that <u>nothing</u> at all has been expressed if a conditional speech act has a false antecedent and is thus "void" (Quine, 1952) . But this cannot be right.

A "void" conditional states no <u>indicative</u> fact. It is about a <u>hypothetical</u> possibility.

In traditional truth table tasks, people are actually shown a false antecedent case. The indicative conditional is then "void". People would not use an indicative form in such a case. They would use a counterfactual.

A void case in more detail

I say to an editor, “If you publish their paper, it will be the worst mistake you ever make.” The editor rejects the paper, but it is published elsewhere to great acclaim.

I can hardly justify myself by claiming that what I said was a material conditional and so true! Equally I cannot claim to have made a “void” utterance in every sense of this term. It was about a hypothetical possibility.

There are corresponding counterfactuals, “If you were to accept the paper, …”, and “If you had accepted it, …”, These can have a high or low probability and be more or less justified.

“Void” indicatives and counterfactuals

“If I am smoking, then I am damaging my health.”

I am clearly not smoking a cigarette. The world is such that the antecedent is false. There is no “ground-floor level” fact that makes this indicative conditional “true”, and I would prefer to assert the counterfactual:

“If I were smoking, then I would be damaging my health.”

The indicative can be called “void” in this sense, but there is also clearly much more to be said about it and its relation to the counterfactual.

Probability and indicative conditionals

"If I take up smoking, my health will improve."

The above conditional, unlike a material conditional, does not become more and more probable as I become more and more determined not to take up smoking.

It is intuitively a highly improbable conditional. But how do we make this judgment?

Ramsey (1929) proposed an intuitive answer to this question, <u>The Ramsey test</u>, which has been immensely influential (see Edgington, 2014, and Evans & Over, 2004).

The Ramsey test for indicatives

People judge *if p then q* by "...adding *p* hypothetically to their stock of knowledge …" and then assessing to what extent *q* follows. They thus fix "... their degree of belief in q given p …" (Ramsey, 1929). The result is that:

$$P(\text{if } p \text{ then } q) = P(q|p)$$

This identity is so important that it has simply been called *the Equation* (Edgington, 1995). In psychology, it would be better to term it *the conditional probability hypothesis*. If it holds, a Bayesian account can be given of people's reasoning in a dynamic process of belief revision and updating.

The Ramsey test: An example

"If I take up smoking, my health will improve."

For the Ramsey test, I suppose hypothetically that I take up smoking, and use my knowledge to assess how likely it is, under that supposition, that my health will improve. The result is a low degree of belief in this conditional.

With that low degree of belief, I decide not to smoke in a Bayesian process of decision making. During this dynamic process, my degree of belief in the conditional stays low.

The Ramsey test for counterfactuals

When *p* is false, Ramsey suggested that *if p then q* becomes "void". But we do make judgments about counterfactuals.

To determine *P(if p then q)*, we suppose that *p* holds, making necessary changes to preserve consistency, and then we assess how likely *q* is.

To apply this extended test to a counterfactual, e.g. "If I had taken up smoking, my health would have improved", I could mentally go back to a time when the antecedent was not yet definitely false, and then use the Ramsey test for indicatives.

Stalnaker's theory: The basics

To evaluate *if p then q*, consider the closest possible world in which *p* is true. Then *if p then q* holds if and only if *q* is true in that world (Stalnaker, 1968, 1970).

Stalnaker was inspired by the Ramsey test in formulating this account of the truth conditions for *if p then q*.

He held that the probability of *if p then q*, P(if p then q), is the conditional probability of *q* given *p*, P(q|p).

Stalnaker's evaluation: "If I take up smoking (p) then my health will improve (q)".

p \ q	1	0
1	1	0
0	0	0

Stalnaker's evaluation: "If I take up smoking (*p*) then my health will not improve (*not-q*)".

q / *p*	1	0
1	1	0
0	1	1

Stalnaker’s theory and probability

Consider the two tables just shown. Stalnaker’s conditional is clearly not truth functional. Now let all four possibilities have equal probabilities: P(p & q) = P(p & not-q) = P(not-p & q) = P(not-p & not-q) = .25

In the first table, P(if p then q) = P(p & q) = .25.

In the second table, P(if p then not-q) = P(not-p or q) = .75.

In both tables, $P(q|p) = .5$.

Lewis's proof: The basic point

Lewis (1976) proved that the probability of a Stalnaker-type conditional cannot, in general, be the conditional probability.

P(q|p) = P(p & q) / P(p), given that P(p) > 0, and note that P(p) = (P(p & q) + P(p & not-q)) .

But for a Stalnaker-type conditional, P(if p then q) depends on the *not-p* states of affairs. This also true for a Lewis-type conditional (Lewis, 1973).

Lewis's proof: Limitation

Lewis assumed that *if p then q* is objectively true or false at every possibility, including the *not-p* possibilities.

But we have already seen reason to hold that *if p then q* may be "void" in *not-p* cases.

By the Ramsey test, *if p then q* has a subjective probability in *not-p* cases, but not an objective truth value. It is in fact striking that the founders of subjective probability theory, Ramsey and de Finetti, had similar views on the probability of conditionals.

The de Finetti table

The "defective" truth table should be called the *de Finetti Table*, or more strictly the *2x2 de Finetti table*.

This table was first proposed by de Finetti in (1936), in his paper on "The logic of probability". This table is part of a probabilistic account of reasoning and the conditional, in which P(if p then q) = P(q|p).

He called this conditional the *conditional event* and related it to a conditional bet, and a conditional bet is basically a bet on a conditional assertion.

The 2x2 de Finetti table for *if p then q*

1 = true, 0 = false, U = uncertain

q / *p*	1	0
1	1	0
0	U	U

The 3x3 general de Finetti table

Baratgin, Over, & Politzer (2013)

Clearly, people do not always know which row of the truth table represents the actual state of affairs.

Which three-value table best represents people's judgments when they are uncertain about *p* or about *q*? Many three-value tables have been proposed. Which best describes the judgments of ordinary people, de Finetti's or another?

The 3x3 de Finetti table

1 = certainly true, 0 = certainly false, U = uncertain

p \ *q*	1	U	0
1	1	U	0
U	U	U	U
0	U	U	U

The Jeffrey (1991) table derived

Much research has confirmed that the probability of *if p then q* is judged to be the conditional probability, P(q|p).

What value replaces the U of “uncertainty” in the table if P(if p then q) = P(q|p)? Jeffrey (1991) showed that:

$$P(q|p) = P(p \,\&\, q)(1) + P(p \,\&\, \text{not-}q)(0) + P(\text{not-}p)U$$

$$P(q|p) = P(p \,\&\, q) + P(\text{not-}p)U$$

$$U = P(q|p)$$

The Jeffrey truth table for *if p then q*

1 = certainly true, 0 = certainly false,

P(q|p) = the conditional probability of *q* given *p*

p \ *q*	1	0
1	1	0
0	P(q|p)	P(q|p)

The Jeffrey table and certain truth

Note that the Jeffrey table implies that a certain conditional has has a value of 1 even if its antecedent is false.

The simplest example is *if p then p* when *p* is false. This has a value of 1 by the Ramsey test, and this value appears in all the consistent cells of the Jeffrey table.

It is a logical relation that makes *if p then p* true, and not a *p & q* fact, when *p* is false. Is there one sense of "true" or many?

We can even say that judgments of subjective taste are "true" - "If you have the fish then you should choose a white wine"

A review and look forward

***The Equation*: P(if p then q) = P(q|p)**

The Equation is often identified with a particular person - Adams, Jeffrey, or Stalnaker.

If the Equation, as the conditional probability hypothesis, is confirmed in experiments, a Bayesian account can be given of when people are coherent in their conditional reasoning, for belief revision and updating.

Some of its deep implications will be covered in the next class.